Ilya Smirnov
Lomonosov Moscow State University, Russia

Mixed Problems for the Telegraph Equation in the Case of a System Consisting of Two Segments with Different Densities and Elasticities

For three mixed problems that describe the longitudinal vibrations of a rod governed by the telegraph equation, expressions for their solutions important for the optimization of boundary controls are obtained in terms of functions involved in the Dirichlet and Neumann boundary conditions. The rod consists of two segments with different densities and elasticities. The control of the force or the displacement is considered at the left and right ends of the rod.

Suppose that the initially stationary rod occupies the segment $0 \leqslant x \leqslant l$ and has the following properties:

- The rod has the linear density $\rho_1 = const$ and Young's modulus $k_1 = const$ on the segment $0 \leqslant x \leqslant \overset{\circ}{x}$.
- The rod has the linear density $\rho_2 = const$ and Young's modulus $k_2 = const$ on the segment $\overset{\circ}{x} \leqslant x \leqslant l$.
- The impedances on these two segments are equal to each other.
- If the segments have different impedances, the point of their contact generates a series of reflected and transmitted (on both sides) waves, which are difficult to take into account completely. However, based on physics considerations, we can assume that this task simplifies considerably when the densities, elasticity coefficients, and lengths of the segments are such that a wave takes identical times to travel over each of them.

We present the analytical form of solutions for these mixed problems.