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Solution to the Navier–Stokes equations with random initial data

We construct a solution to the spatially periodic d-dimensional Navier–Stokes equations with a given distribution of the initial data. The solution takes values in the Sobolev space H^α, where the index $\alpha \in \mathbb{R}$ is fixed arbitrary. The distribution of the initial value is a Gaussian measure on H^α whose parameters depend on α. The Navier–Stokes solution is then a stochastic process verifying the Navier–Stokes equations almost surely. It is obtained as a limit in distribution of solutions to finite-dimensional ODEs which are Galerkin-type approximations for the Navier–Stokes equations. Moreover, the constructed Navier–Stokes solution $U(t,\omega)$ possesses the property:

$$\mathbb{E}[f(U(t,\omega))] = \int_{H^\alpha} f(e^{t\nu\,\Delta}u)\gamma(du),$$

where $f \in L_1(\gamma)$, $e^{t\Delta}$ is the heat semigroup, ν is the viscosity in the Navier–Stokes equations, and γ is distribution of the initial data.

This work generalizes, in several directions, the technique and the results of (1) where the authors prove the existence of the solution to the two-dimensional spatially periodic Euler equations. Unlike (1), our results hold for all viscosities $\nu \geqslant 0$ which includes the Euler ($\nu = 0$) and the Navier–Stokes ($\nu > 0$) cases. They hold for all dimensions $d \geqslant 2$, and for any Sobolev space index $\alpha \in \mathbb{R}$ whereas the result of (1) was proved for $\alpha < -\frac{1}{2}$.

References

(1) S. Albeverio, A.B. Cruzeiro, *Global flows with invariant (Gibbs) measures for Euler and Navier-Stokes Two Dimensional fluids*, Commun. Math. Phys. 129, (1990), 431–444.