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UMD spaces and imaginary powers of operators

S. Guerre-Delabrière (1) proved that if X is a Banach space, then X is UMD (Unconditional Martingale Differences) space if and only if the imaginary power $(-\Delta)^{i\gamma}$, $\gamma \in \mathbb{R}$, of the Laplacian operator on $\mathbb{R}$ ($\Delta = \frac{d^2}{dx^2}$) is bounded from $L^p(\mathbb{R}, X)$ into itself, for every $1 < p < \infty$. In this talk we prove that Guerre-Delabrière's property also holds when the operator $-\Delta$ is replaced by other differential operators (Laguerre, Bessel, Schrödinger,...). If $\mathcal{L}$ denotes he Schrödinger operator $\mathcal{L} = -\Delta + V$ on $\mathbb{R}^n$, where V is a function in $L^1_{\mathrm{loc}}(\mathbb{R}^n)$ satisfying certain reverse Hölder inequality, we prove that a Banach space X is UMD if and only if the imaginary power $\mathcal{L}^{i\gamma}$, $\gamma \in \mathbb{R}$, is bounded from $L^p(\mathbb{R}^n, X)$ into itself, for every $1 < p < \infty$.
This talk is based on joint work with J.J. Betancor, A.J. Castro, J. Curbelo, J.C. Fariña and R. Crescimbeni.

References

(1) S. Guerre-Delabrière, *Some remarks on complex powers of* $(-\Delta)$ *and UMD spaces*, Illinois Math. J. **35** (1991), 401–407.