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Higher order Hardy inequalities (with application to spectral problems)

The Hardy inequality deals with the estimation of the weighted L^q-norm (with weight u) of a function f by the weighted L^p-norm (with weight v) of its derivative f'. To obtain a meaningful inequality, one has to suppose that the function f vanishes somewhere (e.g., using some boundary conditions). In the case of a higher order Hardy inequality, we estimate the norm of the function by the norm of some higher order derivative, and in this case, the number and the form of the corresponding boundary conditions is substantially bigger. The aim of the talk is to show, which of these boundary conditions are reasonable, and for this admissible boundary conditions find conditions on the weight functions u and v (sufficient or necessary and sufficient) which guarantee that the inequality in question holds for all functions f satisfying these boundary conditions.
The problem considered is closely connected with boundary value problems for ordinary differential equations and with weighted inequalities for some integral operators (the so-called Hardy-type inequalities) and has numerous applications, e.g. in the spectral theory of degenerated and/or singular differential operators.
The results presented have been obtained partially in collaboration with K. Kuliev from the University of West Bohemia, Pilsen.