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Global compensated compactness theorem for general differential operators of first order

Let $A_1(x,D)$ and $A_2(x,D)$ be differential operators of the first order acting on l-vector functions $u=(u_1,\cdots,u_l)$ in a bounded domain $\Omega\subset\mathbb{R}^n$ with the smooth boundary $\partial\Omega$. We assume that the H^1-norm $\|u\|_{H^1(\Omega)}$ is equivalent to $\sum_{i=1}^2\|A_iu\|_{L^2(\Omega)}+\|B_1u\|_{H^{\frac{1}{2}}(\partial\Omega)}$ and $\sum_{i=1}^2\|A_iu\|_{L^2(\Omega)}+\|B_2u\|_{H^{\frac{1}{2}}(\partial\Omega)}$, where $B_i=B_i(x,\nu)$ is the trace operator onto $\partial\Omega$ associated with $A_i(x,D)$ for $i=1,2$ which is determined by the Stokes integral formula (ν: unit outer normal to $\partial\Omega$). Furthermore, we impose on A_1 and A_2 a cancellation property such as $A_1A_2'=0$ and $A_2A_1'=0$, where A_i' is the formal adjoint differential operator of $A_i(i=1,2)$. Suppose that $\{u_m\}_{m=1}^\infty$ and $\{v_m\}_{m=1}^\infty$ converge to u and v weakly in $L^2(\Omega)$, respectively. Assume also that $\{A_1u_m\}_{m=1}^\infty$ and $\{A_2v_m\}_{m=1}^\infty$ are bounded in $L^2(\Omega)$. If either $\{B_1u_m\}_{m=1}^\infty$ or $\{B_2v_m\}_{m=1}^\infty$ is bounded in $H^{\frac{1}{2}}(\partial\Omega)$, then it holds that $\int_\Omega u_m\cdot v_m dx\to\int_\Omega u\cdot v dx$. We also discuss a corresponding result on compact Riemannian manifolds with boundary.