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Optimal initial value conditions for local strong solutions of the Navier-Stokes equations in exterior domains

Let u be a weak solution of the Navier-Stokes equations in an exterior domain $\Omega \subseteq \mathbb{R}^3$ and a time interval $[0,T[$, $0 < T \leq \infty$, with initial value u_0 and external force $f = \operatorname{div} F$. Here we address the problem to find the optimal (weakest possible) initial value condition in order to obtain a strong solution $u \in L^s(0,T;L^q(\Omega))$ in some time interval $[0,T[$, $0 < T \leq \infty$, where s,q with $3 < q < \infty$ and $\frac{2}{s} + \frac{3}{q} = 1$ are so-called Serrin exponents. Our main result states, for Serrin exponents s,q with $q \in [\frac{24}{7}, 8]$, a smallness condition on $\int_0^T \|e^{-\nu\tau A}u_0\|_q^s \, d\tau$ to imply existence of a strong solution $u \in L^s(0,T;L^q(\Omega))$; here A denotes the Stokes operator. Moreover, for Serrin exponents s,q with $3 < q < \infty$ we will prove the necessity of the condition $\int_0^\infty \|e^{-\nu\tau A}u_0\|_q^s \, d\tau < \infty$ to get a strong solution u on $[0,T[$, $0 < T \leq \infty$.