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Rearrangement invariant Sobolev spaces on general domains

Suppose Ω is a domain in $\mathbb{R}^n$ having finite Lebesgue measure. Let ρ be a rearrangement invariant (r.i.) functional on the class $\mathfrak{M}_+(\Omega)$ of nonnegative Lebesgue-measurable functions on Ω. Fix $m \in \mathbb{Z}_+$, $1 \leq m \leq n-1$ and let

$$L_\rho^m(\Omega) := \{u : \Omega \longrightarrow \mathbb{R} : \rho(|\bigtriangledown^m u|) < \infty\};$$

here $|\bigtriangledown^m u| := \sum_{|\alpha|=m} \left|\frac{\partial^\alpha u}{\partial x^\alpha}\right|$. We have two interconnected goals. First, seek a functional to describe the smallest set containing rearrangements of functions in $L_\rho^m(\Omega)$. Second, we aim to study refinements of Sobolev-Poincar$\acute{e}$ inequalities which, for spaces with first order derivatives, have the form

$$\inf_{c\in\mathbb{R}} \sigma(u-c) \leq A\rho(|\nabla u|),$$

where σ is another r.i. functional on $\mathfrak{M}_+(\Omega)$.