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Sobolev spaces of differential forms

We study classes of p-integrable k-dimensional differential forms with q-integrable exterior differential. In the case $k = 1$ and $p = q$ these classes $\Omega_{q,p}(M)$ coincide with Sobolev spaces $W_p^1(M)$ on a Riemannian manifold M. The classes $\Omega_{q,p}(M)$ are Lipschitz invariant and together with the exterior differential d form a Banach complex $(\Omega_{q,p}(M), d)$. One of the main topic of this talk will be a detailed analysis of relations between a global cohomology classes of $(\Omega_{q,p}(M), d)$ that we call of $L_{q,p}$-cohomology and a version of Sobolev type inequalities for differential forms. This relation is correct for the classical Sobolev inequality also and corresponds to 1-dimensional $L_{q,p}$-cohomology.
Although the Sobolev type inequalities have interpretations in $L_{q,p}$-cohomology, this will not lead us very far unless we are able to compute some of this cohomology. Unfortunately, this is not an easy task. Some computations of $L_{q,p}$-cohomology for negatively curved Riemannian manifolds and warped products will be discussed.
The classical Sobolev inequality is important because it is a key ingredient in solving partial differential equations. To illustrate this point in the case of the Sobolev inequality for differential forms, we show some applications to quasi-linear equations.