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Morrey type spaces on the base of RIS and BFS

Let $E = E(\mathbb{R}^n)$ be a rearrangement invariant space (RIS) and $\widetilde{E} = \widetilde{E}(\mathbb{R}_+)$ be its presentation on $\mathbb{R}_+ = (0, \infty)$ such that $\|f\|_E = \|f_*\|_E = \|f^*\|_{\widetilde{E}}$. Here f_* and f^* are symmetrical and decreasing rearrangements of the function f, i. e. decreasing functions of the arguments $\rho = |x|,\ x \in \mathbb{R}^n$ or $t \in \mathbb{R}_+$, respectively, equimeasurable with respect to f. Let $F = F(\mathbb{R}_+)$ be a Banach function spaces (BFS). We introduce the local Morrey type space

$$LM_{EF} = LM_{EF}(\mathbb{R}^n) = \left\{ f \in E^{loc} :\ \|f\|_{LM_{EF}} = \left\| \left\| f \chi_{B_r} \right\|_E \right\|_F \right\}$$

Here, $B_r = \{x \in \mathbb{R}^n :\ |x| < r\}$, $|B_r|$ is the measure of the ball and the norm in F is calculated with respect to $r \in \mathbb{R}_+$. In the special case $E = L_p(\mathbb{R}^n),\ F = L_q(w)$ we obtain spaces considered by V. I. Burenkov and M. L. Goldman. Moreover, we introduce the global Morrey type space

$$GM_{EF} = GM_{EF}(\mathbb{R}^n) = \left\{ f \in E^{loc} :\ \|f\|_{GM_{EF}} := \sup \{ \| f(x + \cdot) \|_{LM_{EF}} \ :\ x \in \mathbb{R}^n \} < \infty \right\}.$$

For RIS $E_i = E_i(\mathbb{R}^n),\ i = 1, 2$ consider an operator $A :\ E_1 \to GM_{E_2 F}$ with

$$\|A\|_0 = \sup \{ \|Af\|_{LM_{E_2 F} :\ \|f\|_{E_1} \le 1} \}, \qquad \|A\|_1 = \sup \{ \|Af\|_{GM_{E_2 F} :\ \|f\|_{E_1} \le 1} \} .$$

Proposition 1. *Let A be as above and let*

$$\Psi_A(r) = \sup \{ \|(Af)\, \chi_{B_r}\|_{E_2} :\ \|f\|_{E_1} \le 1 \},\ \ r \in \mathbb{R}_+ .$$

Then the estimates

$$\sup \{ \Psi_A(r)\, \|\chi_{[r,\infty)}\|_F \ :\ r \in \mathbb{R}_+ \} \le \|A\|_0 \le \|\Psi_A\|_F$$

hold.

Corollary: *Let $A :\ E_1 \to E_2$ be bounded. Then,* $\|A\|_0 < \infty \Longleftrightarrow \|\, 1 \,\|_F < \infty$.

Example: *For $F = L_\infty(w)$ we have* $\|A\|_0 = \|\Psi_A\|_F = \sup\, [W\, \Psi_A]$.

Now, $A \in \mathcal{J}(c),\ c \in [1, \infty)$ means that $(Af)_* \le c\, A\, f_*$. Note that for the embedding operator $A = I \in \mathcal{J}(1)$ and that for the maximal operator $A = M \in \mathcal{J}(c)$ for some $c = c(n) \in [1, \infty)$.

Proposition 2. *Let $A \in \mathcal{J}(c)$ and let $\|A\|_* = \sup \{ \left\| \|(AF)_*\, \chi_{B_r}\|_{E_2} \right\|_F \ :\ \|f\|_{E_1} \le 1 \}$. Then we have the estimates*

$$\|A\|_0 \le \|A\|_1 \le \|A\|_* \le c\, \|A\|_0 .$$