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Denting points in Orlicz spaces

Let $S(X)$ (resp. $B(X)$, $B(X)^0$) be the unit sphere (resp. the closed unit ball, the open unit ball) of a real Banach space $(X, \|\cdot\|_X)$. A point $x \in S(X)$ is called

a) an *extreme point of* $B(X)$ (write $x \in \delta_e B(X)$) if for every $y, z \in B(X)$ the equality $2x = y + z$ implies $y = z$.

b) a *strongly extreme point of* $B(X)$ (write $x \in \delta_{se} B(X)$) if for every sequences $(y_n), (z_n)$ in $B(X)$ we have $\|y_n - x\|_X \to 0$ whenever $\|y_n + z_n - 2x\|_X \to 0$.

c) a *denting point of* $B(X)$ (write $x \in \delta_d B(X)$) if $x \notin \overline{co}\left\{B(X) \setminus \left[x + \varepsilon B(X)^0\right]\right\}$ for each $\varepsilon > 0$.

It is well known that

$$\delta_d B(X) \subset \delta_{se} B(X) \subset \delta_e B(X).$$

Let (T, Σ, μ) be a measure space with a σ-finite non-atomic and complete measure μ and $L_0 = L_0(T, \Sigma, \mu)$ be the set of all μ-equivalence classes of real and Σ-measurable functions defined on T. A map $\Phi : \mathbb{R} \to [0, \infty]$ is said to be an *Orlicz function* if it is even, convex, left continuous on the whole $\mathbb{R}^+$, continuous at zero, $\Phi(0) = 0$ and Φ is not identically equal to zero. Given any Orlicz function Φ we define on L_0 the modular I_Φ by

$$I_\Phi(x) = \int_T \Phi(x(t))\, d\mu.$$

By the *Orlicz function space* L_Φ we mean the set of all $x \in L_0$ such that $I_\Phi(cx) < \infty$ for some $c > 0$. The space L_Φ is equipped with the so called *Orlicz norm* that can be expressed by the following Amemiya formula

$$\|x\|_\Phi = \inf_{k>0} \frac{1}{k}\left(1 + I_\Phi(kx)\right).$$

We will show a necessary conditions for a point x from the unit sphere to be a denting point of the unit ball of Orlicz spaces generated by arbitrary Orlicz functions and we will show that strongly extreme points need not be denting points in Orlicz spaces. Moreover, we give examples of Orlicz spaces with unit ball without denting points.

This is joint work with Ryszard Płuciennik from Poznań University of Technology.