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Optimal reconstruction of derivatives of functions belongs to the Sobolev space

The problem of reconstructing the derivatives important as the concept of derivative (see, for example, (1-3)).For positive real r the Sobolev class $W_2^r(0,1)^s$ is the set of all integrable function $f(x) = f(x_1, ..., x_s)$ that are 1-periodic in each variable and, with trigonometric Fourier-Lebesgue coefficients satisfy the condition

$$\sum_{(m_1,...,m_s)\in Z^s} \left|\hat{f}(m_1,...,m_s)\right|^2 \left(\overline{m}_1^{2r} + ... + \overline{m}_s^{2r}\right) \le 1,$$

where Z^s set of all vectors $m = (m_1, ..., m_s)$ with integer components, $\overline{m}_j = \max\{1; |m_j|\}$ $(j = 1, .., s)$.

Theorem. *Let integer* $s\,(s = 2, 3, ...)$*, real numbers* $\alpha_j \ge 0\,(j = 1, \ldots, s)$ *and* r *such, that* $r > \big(\sum\limits_{j=1}^{s} \alpha_j + \frac{1}{2}\big)s$*. Then holds (derivatives in the sense of Weyl)*

$$\sup_{f\in W_2^r(0,1)^s} \left\|f^{(\alpha_1,...,\alpha_s)}(x) - \varphi_N(f(\xi_1), ..., f(\xi_N); x)\right\|_{L^2[0,1]^s} \underset{s,r,\alpha_1,...,\alpha_s}{\succ\prec} N^{-\frac{r-(\alpha_1+...+\alpha_s)}{s}} \; (p = 2, 3, ...;\ N = p^s),$$

where $(2\pi i m_j)^{\alpha_j} = (2\pi |m_j|)^{\alpha_j} e^{i\alpha_j\left(\mathrm{sgn}(m_j)\frac{\pi}{2}\right)}$,

$$\varphi_N(f(\xi_1), ..., f(\xi_N); x) = \frac{1}{N} \sum_{\xi^{(n)}\in B_N} f\left(\xi^{(n)}\right) \sum_{\substack{|m_j| < \frac{p}{2} \\ j = 1, 2, ..., s}} (2\pi i m_1)^{\alpha_1} ... (2\pi i m_s)^{\alpha_s} \hat{f}(m)\, e^{2\pi i\left(m, x-\xi^{(n)}\right)}$$

and $B_N = \left\{\xi^{(n)} = \left(\frac{n_1}{p}, ..., \frac{n_s}{p}\right) :\ n = (n_1, ..., n_s) \in Z^s,\ 0 \le n_j < p,\ j = 1, 2, ..., s\right\}$.

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