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On Weyl numbers of Sobolev embeddings of weighted function spaces

We investigate an asymptotic behaviour of Weyl numbers x_k of compact embeddings of weighted Besov spaces. The weights of polynomial growth at infinity $w_\alpha(x) = (1+|x|^2)^{\frac{\alpha}{2}}$ as well as weights of logarithmic growth at infinity $v_\alpha(x) = (1+\log(e+|x|^2))^\alpha$ are considered, $\alpha > 0$. Let $\delta = s_1 - s_2 - d(\frac{1}{p_1} - \frac{1}{p_2})$ and $\frac{1}{t} = \frac{1}{p_2} - \frac{1}{p_1}$. We prove that:

(i) if $\min(\alpha,\delta) > d\max(\frac{1}{p_2} - \frac{1}{p_1}, 0)$, then

$$x_k(B^{s_1}_{p_1,q_1}(\mathbb{R}^d, w_\alpha) \hookrightarrow B^{s_2}_{p_2,q_2}(\mathbb{R}^d)) \ \sim\ k^{-\beta}, \quad \text{with} \quad \beta = \begin{cases} \frac{\min(\alpha,\delta)}{d}, & 2 \le p_1 \le p_2 \le \infty, \\ \frac{\min(\alpha,\delta)}{d} - \frac{1}{t}, & 1 \le p_1 \le p_2 \le 2, \\ \frac{\min(\alpha,\delta)}{d} - \frac{1}{2} + \frac{1}{p_1}, & 1 \le p_1 < 2 < p_2 \le \infty, \\ \max(1, \frac{p_1}{2})\frac{\min(\alpha,\delta)}{d} - \frac{1}{t}, & 1 \le p_2 < p_1 \le \infty \\ & \text{and } \min(\alpha,\delta) > \frac{1}{t}. \end{cases}$$

(ii) if $\delta > 0$, then

$$x_k(B^{s_1}_{p_1,q_1}(\mathbb{R}^d, v_\alpha) \hookrightarrow B^{s_2}_{p_2,q_2}(\mathbb{R}^d)) \ \sim\ (1+\log k)^{-\alpha} k^{-\beta}, \quad \text{with } \beta = \begin{cases} \frac{1}{p_1} - \frac{1}{p_2}, & 1 \le p_1 \le p_2 \le 2, \\ \frac{1}{p_1} - \frac{1}{2}, & 1 \le p_1 < 2 < p_2 \le \infty, \\ 0, & 2 \le p_1 \le p_2 \le \infty. \end{cases}$$