Maximal operators of Fourier multipliers

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Motivation: Maximal Bochner-Riesz

Bochner-Riesz

$$m_\alpha = (1 - |\xi|^2)_+^\alpha$$

$$B_R^\alpha f = \mathcal{F}^{-1}(m_\alpha(\cdot/R)\hat{f})$$

Well known theory.

Maximal Bochner-Riesz

$$M_\alpha f = \sup_R |B_R^\alpha f|$$

Motivation: Maximal hyperbolic Bochner-Riesz

Hyperbolic Bochner-Riesz

On $\mathbb{R}^2$

$$m^{\alpha}(\xi_1, \xi_2) = ((1 - (\xi_1 \xi_2)^2)_+^{\alpha}$$

$$T_R^{\alpha} f = \mathcal{F}^{-1}(m^{\alpha}(\cdot/R)\hat{f})$$

The L^p boundedness of T^{α} was studied by El-Kohen and Carbery (full interplay p, α).

Maximal hyperbolic Bochner-Riesz

$$M^{\alpha} f = \sup_R |T_R^{\alpha} f|$$

Open problem: is there any α, p such that M^{α} is bounded on L^p?

Comparison of the operators

Bochner-Riesz
Homogeneous convolution kernel

Maximal Bochner-Riesz
Maximal operator for the "core" part, other techniques (square functions etc.) for the rest

Hyperbolic Bochner-Riesz
Marcinkiewicz multiplier structure

Maximal hyperbolic Bochner-Riesz
The "core" is Marcinkiewicz multiplier, no maximal estimate aviable, crucial problem

Marcinkiewicz theorem

Marcinkiewicz multiplier

$$|\partial_1 \partial_2 m(\xi_1, \xi_2)| \leq A|\xi_1|^{-1}|\xi_2|^{-1}$$

Marcinkiewicz Theorem

Supose that m is a Marcinkiewicz multiplier. Then the operator

$$T^m f = \mathcal{F}^{-1}(m(\cdot)\hat{f})$$

is L^p bounded for $1 < p < \infty$.

Maximal Marcinkiewicz theorem

Maximal Marcinkiewicz Theorem

Supose that $m_1, \dots, m_N$ are Marcinkiewicz multipliers with an uniform constant A. Then

$$M_N f(x) = \sup |T^{m_i} f|(x)$$

is bounded with constant $C_p A(\log N)$. The logarithmic dependence on N cannot be improved.

Also holds in dimension n with constant $C_{p,n} A(\log N)^{n/2}$.

Maximal Mikhlin-Hörmander theorem

Mikhlin-Hörmander multiplier

On $\mathbb{R}^n$

$$|\partial^\beta m(\xi)| \leq A|\xi|^{-|\beta|}$$

for all $|\beta| \leq n$.

Maximal Mikhlin-Hörmander theorem

Supose that $m_1, \ldots, m_N$ are Mikhlin-Hörmander multipliers, then

$$M_N f(x) = \sup |T^{m_i} f|(x)$$

is bounded with constant $C_p C_n A(\log N)^{1/2}$. The power of the logarithm cannot be improved.

(Grafakos, Honzik, Seeger 2004)