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COHOMOLOGY OF MODULI SPACES IN DIFFERENTIAL OPERATORS CLASSIFICATION TO THE FIELD THEORY

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Abstract. *We consider a generalization of the Radon-Schmid transform on coherent D-modules of sheaves of holomorphic complex bundles inside a cohomological context, with the purpose of establishing the equivalences among geometric objects (vector bundles), and algebraic objects as they are it the coherent D-modules, these last with the goal of establish a classification of the differential operators through the connections of the holomorphic complex bundles. The class of these equivalences conform a moduli space on coherent sheaf's that define solutions in field theory. Also by this way, and using the Penrose transform in the context of coherent D-modules we find conformal classes of space-time. The space of isomorphisms obtained by these classes is equal to an important Moduli space in many problems of field theory.*

Key words: Integral operators cohomology, moduli space, cohomology of cycles, integral curvature.

Nomenclature

$\Box$.- Wave operator. This is a differential operator that composes the wave equation on the space $\mathbb{R}^4$, also called D'Alambert operator.

$SL(4, \mathbf{C})$.- Special linear group of the complex matrices of rank four. Have the structure of Lie group.

$\mathfrak{P}$.- Penrose transform operator.

D_X.- Is the sheaf of rings of holomorphic linear differential operators (a D-module).

O_X.- Sheaf of holomorphic functions on a complex manifold X.

$M_{qc}(D)$.- Category of D-modules.

$M_{coh}(D)$.- Category of D-modules that is subcategory of $M_{qc}(D)$.

$\mathfrak{E}_X$.- Ring naturally endowed with a $\mathbb{Z}$-filtration by the degree.

$D\mathcal{M}$ - Coherent D_X-module. This D-module is the fundamental ingredient of the equivalences of the objects of moduli space.

$\mathcal{M}$.- D_X-module object of our research and their category is the useful to realize the classification of linear differential operators in field theory. This D-module represent a system of PDE, in mathematical physics to which it is necessary to give solution.

$\mathcal{G}$.- Coherent Sheaf of the D_X-module $\mathcal{M}$.

D_Y.- Image of D_X-module under Penrose transform.

$D\mathscr{L}$.- D_X-module of bundles of lines.

$O_{\mathbb{P}}(k)$.- Coherent Sheaf of D-modules that are $D_{\mathbb{P}}$-modules, and that cans be induced to $D_{\mathbb{F}}$-modules.

SUSY Cycles.-Super-symmetry Cycles.

QFT.- Quantum field theory.

Introduction

Is opportune consider a generalization of the Radon-schmid [28], on coherent D-modules [7], of a sheaf of holomorphic bundles into of a cohomological context [16], with the goal of establish the equivalences between geometrical objects (vector bundles), and algebraic objects since those are the coherent D-modules, this last with the goal establish the conformal classes useful to define adequately the differential operators that defines the field equations of all the microscopic and macroscopic phenomena of the space-time through the connections or operators of the form [5], of the complex holomorphic bundles in gauge theory. The class of these equivalences is precisely our moduli space, the which having in consideration the differential operator that define the connection of the corresponding vector bundle establish the relation among dimensions of cohomological classes [10], and the vector bundles corresponding to the differential operators of the equations of form (*connection*), of the Riemannian manifold [10]. But studies in algebraic geometry has established that manifolds of Calabi-Yau have the integrals of type Penrose, or at least complex integrals on strings like equivalences of geometrical invariants under the philosophy of the mirror symmetry. From a point of view of coherent D-modules, this conformally comes established in automatic form through of the use of Penrose transforms on D-modules. Of this manner we arrive to D-strings and D-branes corresponding to specific classes of derived sheaves obtained for the appropriate generalized Penrose transform [20]. We define to our moduli space like the space of the invariant differential operators of the $\overline{\partial}$-cohomology modulus that are conformally invariants. We determine a cohomology between moduli spaces on the space of differential operators that accepts a scheme of integral operators cohomology of Penrose type in the context of the coherent D-modules [36], since the scheme of the irreducible unitary representations to these operators are unitary representations of compact components of the group $SL(4,\mathbf{C})$ [25].

1. Conformally Invariant Operators, Penrose Transform and Derived Sheaves

Exists inkling of the class of differential operators that accept an reinterpretation of an integral operators cohomology (as the due for the Penrose transform, the Twistor transform, etc), is accordingly it is have the class of invariant differential operators, of fact the Penrose transform generates these conformal invariants operators [5], and thus we can identify the conformal classes to the which belongs [18]. Some examples of these differential operators are the existent for the massless field equations (to flat versions and curved of some of their similar ones [14]), and the conformal invariant wave operator given by the map [5]:

$$\Box + R/6 : O[-1] \longrightarrow O[-3] \tag{1.1}$$

or also Einstein's operator [5]:

$$\nabla_{(A(A'}\nabla_{B)B')} + \Phi_{(AB)(A'B')} : \Box[-1] \longrightarrow \Box_{(AB)(A'B')}[-1] \tag{1.2}$$

or the conformal invariant modification of the square of the wave operator $O[-1] \longrightarrow O[-4]$, that is to say; the wave operator that involves in terms of Ricci tensor [5],

$$\Box^2 : \phi \longrightarrow \nabla_b[\nabla_b\nabla_a - 2R^{ab} + ((2/3)Rg^{ab})\nabla_b] \tag{1.3}$$

Then the integration of the partial differential equations corresponding to these differential linear invariant operators is carried out as integrals transform of the Penrose type, because the context of the irreducible unitary representations for these operators are unitary representations of compact components of the group $SL(4,\mathbf{C})$, such as $SO(2n)$, [25], of fact in the flat case the classification of differential invariant operators like those described previously are a problem of representations theory of Lie groups applied to the group of $SL(4,\mathbf{C})$ and its compact subgroups [25]. Then one visualizes to these operators through of Lie group $SL(4,\mathbf{C})$, as equi-variants operators among homogeneous vector bundles on M considering to $SL(4,\mathbf{C})$, like homogeneous space [10], [34]. The integral operators in this case are realizations of these representations and they are orbital integrals of one integral transform from the resolutions to these differential equations which in this concrete case are of Penrose type. The problem is solved using the machinery of representations theory and is given in [36] and with more generality in [31]. In after studies, the local twistor connection [32] is used to investigate the questions as to whether these operators have curved analogues *i.e.* conformally invariant operators in the curved case. For example, some of these operators that are mentioned in (1.2) and (1.3), are included in a conformal class that is obtained by the image of the Penrose transform on the corresponding sheaf whose germs are these differential operators in the holomorphic vector bundle given. *The Penrose transform generates conformally differential operators.* We consider the Penrose transformation, [6] through of the correspondence

$$\begin{array}{ccc} & \mathbb{F} & \\ {\scriptstyle \upsilon}\swarrow & & \searrow{\scriptstyle \pi} \\ \mathbb{P} & & \mathbb{M} \end{array} \tag{1.4}$$

where $\mathbb{F} = \mathbb{F}_{1,\,2}(V)$, is the manifold of flags of dimension one and two, associated to 4-dimensional complex vector V. Be $\mathbb{P} = \mathbb{F}_1(V)$, such that $\mathbb{F}_1(V) \cong \mathbb{P}^3(\mathbb{C})$, (complex lines in $\mathbb{C}^4$), and be $\mathbb{M} = \mathbb{F}_2(V)$, such that $\mathbb{F}_2(V) \cong G_{2,\,4}(\mathbb{C})$, (Grassmannian manifold of 2-dimensional complex subspaces), con $\mathbb{M} \cong \mathbb{R}^4 \otimes_{\mathbb{R}} \mathbb{C}$, where

$$\mathbb{M} = \left\{ \underline{z} \in \mathbb{C}^4 | \underline{z} = (z_1, z_2, z_3, z_4), \forall z_i = x_i + jy_i, \forall\ x_i, y_i \in \mathbb{R} \right\} \tag{1.5}$$

is the 4-*dimensional complex compactified Minkowski space* [33]. The projections of $\mathbb{F}$, are given for:

$$v(L_1, L_2) = L_1 \tag{1.6}$$

and

$$\pi(L_1, L_2) = L_2 \tag{1.7}$$

where $L_1 \subset L_2 \subset V$, are complex subspaces of dimension one and two, respectively, defining an element (L_1, L_2), of $\mathbb{F}$, to know

$$\mathbb{F} = \{(L_1, L_2) \in V \times V | L_1 \subset L_2 \subset V, v(L_1, L_2) = \pi(L_1, L_2) = L_2\} \tag{1.8}$$

If $\mathbb{M}$, is of compactified Minkowski [33] then

$$\begin{aligned} &\{set\ of\ equations\ of\ massless\ fields\} \cong \\ &\cong \left\{ dF = 0, dF^* = j, W \circ \delta = 0, R^{ij} = 0, R^{ij} - g^{ij} R = 0... \right\}, [31] \end{aligned} \tag{1.9}$$

is to say, it is require the *spectral resolution of complex sheaves* [33], of certain class seated in the *projective space* $\mathbb{P}$, to give solution to the field equations modulo a flat conformally connection [33], [8].

$$O^{o}_{\mathfrak{p}}(h) \longrightarrow ... \longrightarrow O^{i}_{\mathfrak{p}}(h) \longrightarrow O^{i+1}_{\mathfrak{p}}(h) \longrightarrow ... \longrightarrow 0 \tag{1.10}$$

Be $\mathfrak{p}$, the *Penrose transform* [33], *associated to the double fibration* in (1.4), used to represent the holomorphic solutions of the *generalized wave equation* [33], with parameter of *helicity* h [6], [22]:

$$\Box_h \phi = 0 \tag{1.11}$$

on some open subsets $U \subset \mathbb{M}$, in terms of *cohomological classes of bundles of lines* [33], on $\underline{U} = v(\pi^{-1}(U)) \subset \mathbb{P}$ ($\mathbb{P}$, is the *super-projective space*). Is necessary to mention that *these cohomological classes* are the conformal classes that it is wants determine to solve *the phenomenology of the space-time* to diverse interactions studied in *gauge theory* [3] and can construct a general solution of the general cohomological problem of the *space-time*. With major precision we establish that bundle of lines on $\mathbb{P}$, are given $\forall\ k \in \mathbb{Z}$, by the kth-tensor power $O_{\mathbb{P}}(k)$, of the tautological bundle [17], (is the bundle that serve to explain in the context of the bundles of lines on $\mathbb{P}$, the general bundle of lines of $\mathbb{M}$). Be $h(k) = (1 + k/2)$, $\forall\ x \in \mathbb{P}$, and $\underline{x} = \pi(v^{-1}(x))$. Then a result that establish the *equivalences* on the cohomological classes of the bundle of lines on $\underline{U}$, and the family of solutions of the equations (1.9), (equations of the massless fields family on the Minkowski space $\mathbb{M}$, with helicity h), is the given by:

Theorem 1.1. ***(with classic Penrose transform)*** *Be $U \subset \mathbb{M}$, an open subset such that $U \cap \underline{x}$, is connect and simply connect $\forall\ x \in \underline{U}$. Then $\forall\ k < 0$, the associated morphism to the twistor correspondence (1.4); the which map a 1-form on $\underline{U}$, to the integral to along of the fibers of π, of their inverse image for v, induce an isomorphism of cohomological classes:*

(1.12) $$H^1(\underline{U}, O_{\mathbb{P}}(k)) \cong ker(U, \Box_{h(k)})$$

Proof. [33] □

Which are the classes that are extensions *of the space of equivalences of the type* (1.12), Why are necessary these classes to include more phenomena of the space-time?, What version of the *Penrose transform* will be required?. Part of the object of our research is centered in *the extension of the space of equivalences of the type* (1.12), under a more general context given through of the language of the D-modules, is to say, we want *extend our classification of differential operators of the field equations* to context of the *G-invariant holomorphic bundles* and obtain a complete classification of all the differential operators on *the curved analogous of the Minkowski space of* $\mathbb{M}$. Thus our moduli space will be those of *the equivalences of the conformal classes given in* (1.12), in the language of the *algebraic objects D-modules with coefficients in a coherent sheaf* [31]. The following research is based in the scheme of implications (*see figure 1*).

A way of answering the first and second ask that it is to analyze the origin of the structure of the complexes that define the microscopic physical phenomena. Across this way and in natural form we can establish equivalence (*isomorphisms*), between derivative categories and categories of physical phenomena, considering a complex like defined in (1.10), for the micro-local context in which a regularity theorem subjacent for the Penroses transform on D-Modules, that is to say, given a D-Modules complex [31], [1]

(1.13) $$\ldots \longrightarrow \mathfrak{E}_0 \longrightarrow \mathfrak{E}_1 \longrightarrow \mathfrak{E}_2 \ldots$$

we can map it to a system of branes/anti-branes, in which every $\mathfrak{E}_i, \forall i$, odd it defines branes and other sheaves define anti-branes. Such D-branes and anti-D-branes they are defined of equal form in the space-time, although dynamically they annihilates for defining a pair of particle and anti-particle [1]. This bears to the application of a symplectic context, which also is a consequence of the foliation in Lagrangian submanifolds of $\mathbb{M}$, inside the problem of the unity of the Radon-Penrose transform on the same coherent D-Modules [36].

2. Some Differential Operators Classifications

2.1. Classification of Homogeneous Bundles on Projective Spaces of Orbits of Lie Group $SL(4, \mathbf{C})$.

2.1.1. *In Complex Minkowski Space:* The integral of line of Penrose in integral geometry has the interpretation like it was mentioned of one Radon transform [10], it has more than enough lines of a flag manifold $\mathbb{F} = \{L | L \subset \mathbb{C}_4\}$. Somehow all these integrals belong to oneself cohomology class which can be determined calculating the cohomology of μ^{-1}O(p, q, r), using the spectral succession

(2.1) $$E_{p,q} = \Gamma(U, v_*^q(\mathbb{P}^P))$$

Then it is possible to calculate the cohomology groups $H^{p+q}(U', \mu^{-1}O(p, q, r))$, in terms of $H^{p+q}(U', (\mathbb{P}^p))$, based on the meaning of $E_{p,q}$. These cohomology groups are sections of the sheaf with coefficients on the make fibred $O(p, q, r)$. The space

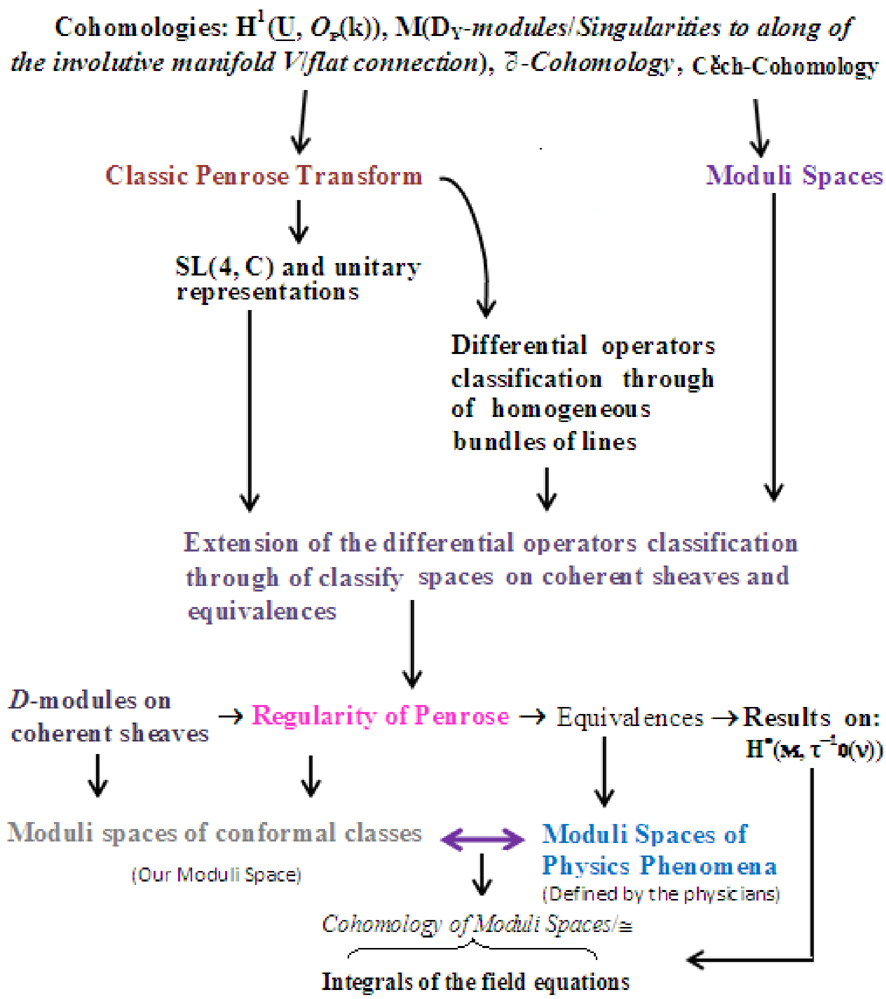

FIGURE 1

$\mu^{-1}O(p,q,r)$, it represents the inverse image of this sheaf. $U \subset \mathbb{M}, U' = v^{-1}(U) \subset \mathbb{F}$ and $\mu(U') \subset \square$, and they are the corresponding homomorphisms of the double fibration in integral geometry to relate objects in $\mathbb{M}$, $\mathbb{F}$ and $\mathbb{P}$ where $\mathbb{P}$ can be a projective twistor space and $\mathbb{F}$ the flags manifold. The resolution is

$$0 \to \mu^{-1}(V) \to \mathbb{P}^*. \tag{2.2}$$

Of $\mu^{-1}(V)$ is given by vector bundles, where the maps $\mathbb{P}^i \leftarrow \mathbb{P}^{i+1}$, are differential operators

2.1.2. *In Analogue Curved Spaces:* (*see table 1*). The n-th direct image sheaves, $v^2 * O(p,r)[q]$ over $\mathbb{M}$, are defined by:

$$\Gamma(U, v^2 * O(p,q)[q]) = H^n(U', O(p,r)[q]) \tag{2.3}$$

Table 1. Analogue Curved Spaces.

Resolution
$0 \longrightarrow \mu^{-1}O(p,q,r) \longrightarrow O(p,r)[q] \overset{D^{q+1}}{\rightarrow} O(p+q+1, r+q+1)[-q-2] \longrightarrow 0$
Differential Operators
$D^1 = \nabla$ $D^2 = \nabla^2 + \Phi$ $D^3 = \nabla^3 + 4\Phi\nabla + 2(\nabla\Phi)$ $D^4 = \nabla^4 + 10\Phi\nabla^2 + 10(\nabla\Phi)\nabla + 3(\nabla^2\Phi + 3\Phi^2)$
Direct Images
$p \geq 0, r \geq 0;\ v^0 * O(p,r)[q] = O^{\overbrace{(A'\ldots L')}^{p}\overbrace{(A\ldots L)}^{r}}[q]$ $p \leq -2, r \geq 0;\ v^1 * O(p,r)[q] = O^{\overbrace{(A'\ldots L')}^{-p-2}\overbrace{(A\ldots L)}^{r}}[p+q+1]$ $r \leq -2, p \geq 0;\ v^1 * O(p,r)[q] = O^{\overbrace{(A'\ldots L')}^{p}\overbrace{(A\ldots L)}^{-r-2}}[r+q+1]$ $p \leq -2, r \leq -2;\ v^2 * O(p,r)[q] = O^{\overbrace{(A'\ldots L')}^{-p-2}\overbrace{(A\ldots L)}^{-r-2}}[p+q+r+2]$
All others vanish

For examples.

(1) Consider the resolution on Γ, through of double fibration $\mathbb{P} \leftarrow \mathbb{F} \rightarrow \mathbb{M}$:
$0 \longrightarrow \mu^{-1}O(-4,0,0) \longrightarrow O(-4,0)[0] \overset{D_1}{\rightarrow} O(-3,1)[-2] \rightarrow 0$, Then the cohomological space given by $H^1(U'', O(-4,0,0))$, is kernel of a first order differential operator from $O_{(A'B')}[-1]$ to $O_{(A'A')}[-2]$.

(2) Consider the resolution.
$0 \longrightarrow \mu^{-1}O(0,1,0) \longrightarrow O(0,0)[1] \overset{D_2}{\rightarrow} O(2,2)[-4] \rightarrow 0$, and thus immediately see that $H_0(U'', O(0,1,0))$, is isomorphic to the kernel of a second order conformally invariant differential operator from $O[1]$ to $O_{(AB)}$ $_{(A'B')}[-2]$. Then to D_2 it is have

$$H^0(U'', O(0,1,0)) = \{\sigma \in \Gamma(U, O[1]) : (\nabla + \Phi)\sigma = 0\} \tag{2.4}$$

The flat space invariant operators fall into two classes, standard and non-standard. The standard operators are those which are obtained as direct images of the operators D^n, from $\mathbb{G} = F_{1,2,3}(\mathbb{C}4)$, the flat space version of S (total space of bundle of null directions over a complex conformal spin 4-manifold $\mathbb{M}$), or as compositions of the same. The non-standard operators are basically those involving the wave operator in some way (for example the classification obtained by the Penrose transform on D-modules). They are also distinguished by the fact that they appear when the spectral sequence for the transform from $\mathbb{P}$, fails to converge at the first step. Since one has seen in the beginning of the *section 2*, the standard operators all have curved analogues which can be generated as direct images of D^n, from S, given by (1.13), and it is noteworthy that the correction terms involve only the

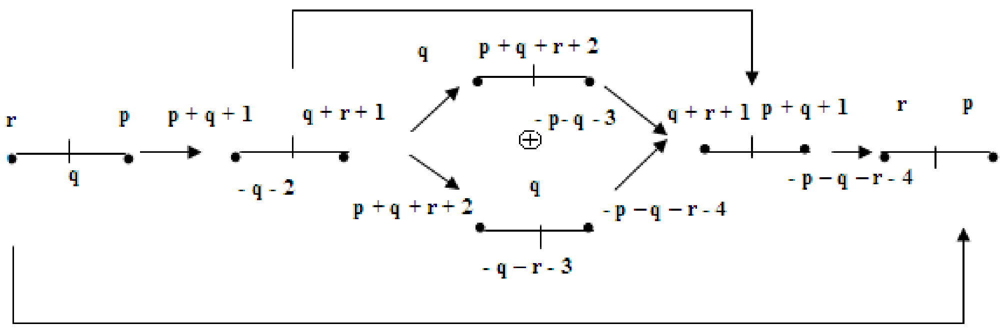

Figure 2

trace-free part of the Ricci curvature. It would seem at present that some non-standard operators fail to have curved analogues (for example: $\nabla^{n/2} : \Lambda^0 \to \Lambda^1$). Of relevance to arguments in [19], but of more interest in the problem of generate and classify conformally invariant differential operators is the BGG resolution on complex Minkowski space $\mathbb{M}$. The diagrams deduced for this resolution are diagrams of bundles and differential operators in $\mathbb{M}$, are given by (*see figure 2*), where we use the notation $\overset{p}{\bullet}\!-\!\overset{q}{|}\!-\!\overset{r}{\bullet} = O^{\overbrace{A'\ldots L'}^{p}\overbrace{A\ldots L}^{r}}[q]$

with some of the numbers below the diagram for reasons of space. We note that this only makes sense provided $p, r \geq 0$, and when this convention is violated, the entry should be replaced by the zero sheaf.

3. Resolutions on D-Modules

Consider the category of D-modules given by space $M_{qc}^{L}(D)$, it such that the category of quasi-coherent left D-modules on X, is isomorphic to the category $M_{qc}^{L}(D)$, of quasi-coherent right D^{opp}-modules on X. For a category $M_{qc}(D)$, of D-modules we denote by $M_{coh}(D)$, the corresponding subcategory of coherent D-modules. We consider a correspondence

$$X \xleftarrow{v} S \xrightarrow{\pi} Y \tag{3.1}$$

where all the manifolds are analytic and complex, v and π, are proper and (v, π), induces a closed embedding $S \hookrightarrow X \times Y$ [36], Be $dS = dim_C S$, with $d_{S/Y} = d_S - d_Y$. We define the transform of a sheaf F on X, (more generally, of an object of the derived category of sheaves), like

$$\Phi_S F = R\pi v^{-1} F[d_{S/Y}] \tag{3.2}$$

and we define the transform of a D_X-module $\mathcal{M}$, like $\underline{\Phi}_S \mathcal{M} = \underline{\pi}_* D v^* \mathcal{M}$, where $\underline{\pi}_*$ and v^* denotes the direct and inverse images of π and v, respectively, in the sense of the D-modules[1] [7], and we consider also [1]

$$\Phi_S \mathcal{G} = Rv!\pi^{-1}\mathcal{G}[d_{S/X}] \tag{3.3}$$

to a sheaf $\mathcal{G}$, on Y. Then it is have the formula

$$\Phi_S RHom_{D_X}(\mathcal{M}, O_X) = RHom_{D_Y}(\underline{\Phi}_S\mathcal{M}, O_Y) \tag{3.4}$$

of the which it is deduce the formula, to $\mathcal{G}$, the sheaf on Y, (coherent sheaf):

(3.5)

$$R\Gamma(X; RHom_{D_X}(\mathcal{M} \otimes \Phi_S\mathcal{G}, O_X)[d_X] \cong R\Gamma(Y; RHom_{D_Y}(\underline{\Phi}_S\mathcal{M} \otimes \mathcal{G}, O_Y)[d_Y]$$

This define a categorical equivalence of the transformation (1.4), in the context of the *right derived D-modules*, $D^b(\mathcal{M}^R_{qc}(D^v))$, because is necessary to give an equivalence with a sub-category of the *right* D-modules that have support in Y, to of that form guarantee the inverse image of $\mathfrak{p}$, and with it to obtain an image of closed range of the operator $\mathfrak{p}$, conformed their uniquely on the given context [6], [17].

Theorem* 3.1.** ***(Theorem of Kashiwara) *Be $i : Y \longrightarrow X$, of the correspondence (3.1), a closed immersion. Then the direct image functor i_+, is an equivalence of $M^R_{qc}(D^i)$, with full sub-category of $M^R_{qc}(D)$, consisting of modules with support in Y.*

Proof. [20]. □

This equivalence preserves *coherency and holonomicity* [4]. Then preserve *conformability* in $\mathbb{M}$ [31], [14]. We formulate in the language of the D-modules and their sheaves, like the given in a resolution (1.10), the corresponding between the space of coherent D-modules and the space of equations of massless fields. This can establish it if we can grantee the unity of the Penrose transform. But for it is necessary to include a result that establishes the regularity in the analytical sense of the Riemannian manifold, which shapes the space-time, and that allows the application of the involutive distribution theorem on integral submanifolds as solutions of the corresponding equations of field without mass on submanifolds isomorphic in the Khalerian model given in the *Flat model* given on $G_{2,4}(\mathbb{C})$. Of fact an analogy in the obtaining of models of space-time (under the same reasoning), must be realized between *special Lagrangian submanifolds* and m-folds of Calabi-Yau. But to it we need to define the complex micro-local structure that define all the phenomena of strings and branes in microscopic level, that they happen in the 6-dimensional component of the universe (*6-dimensional compact Riemannian manifold*), with ratio of the order $10^{-33}cm$ (Max Plank longitude of a string). The *Penrose transform* is an integral geometric method of interpreting elements of various analytic cohomology groups on open subsets of complex projective 3-space as solutions of linear differential equations on the Grassmannian of 2-planes in a 4-space. The motivation for such transform comes from interpreting of this Grassmannian as the complexification of the conformal compactification of the Minkowski space, the

[1] To define the images of direct functors to D-modules it is have that use derived categories. For it, is simpler defining them for right D-modules. Be $D^b[\mathcal{M}^R_{qc}(D^v)]$, the derived category bounded for right quasi-coherent D^v- modules of the form: $R_{v+}(\mathscr{V}) = Rv^*(\mathscr{V} \otimes^R_D vD_{Y\longrightarrow X})$ where $\mathscr{V}$ is the characteristic manifold and R, is the *right derived functor* following: $R : \mathcal{M}^R_{qc}(D^V) \rightarrow \mathcal{M}_{gc}(D)$ Also $r : Y \rightarrow X$, and $D_{Y\rightarrow X} = v^*(D) = O_Y \otimes_{v-1} (OX)v^{-1}D$. Then $D_{Y\rightarrow X}$, is a right v^{-1} D-modulo to the right multiplication in the second functor.

differential equations follows being the massless field equations of various helicities. By the Kashiwara theorem (*theorem 3.1*), and some results of Oshima [3], [21], on involutive manifolds [37], we can characterize certain spaces to the regularity of the images of $\mathfrak{p}$, in D-modules.

4. A Micro-Local Analysis and Version of the Penrose Transform

Let $V \subset \dot{T}^*X$, be a conic regular involutive submanifold. We say that a coherent D^X-module $\mathcal{M}$, has regular singularities on V, if so has $\mathfrak{E}\mathcal{M}$ ($\mathcal{M}$ is regular if $\mathfrak{E}\mathcal{M} \subset \mathcal{M}$).We denote by $Mod_{RS(V)}(D_X)$, the thick subcategory of $Mod_{good}(D_X)$, whose objects have regular singularities on V, and by $D^b_{RS(V)}(D_X)$, the full triangulated subcategory of $D^b_{good}(D_X)$, whose objects have cohomology groups belonging to $Mod_{RS(V)}(D_X)$:

$D^b_{RS(V)}(D_X) \nearrow^{Mod_{good}(D_X),\supset Mod_{RS(V)}(D_X)} \hookrightarrow \nwarrow D^b_{good}(D_X)$ Associated to the correspondence given in (3.1), we give the micro-local correspondence:

$$\begin{array}{ccccc} & & \Lambda & & \\ p_{1|\Lambda} & \swarrow & & \searrow & p_2^a|_\Lambda \\ \dot{T}^*X & & & & \dot{T}^*Y \end{array} \tag{4.1}$$

The manifold Λ, being Lagrangian, is well acquaintance that $p_{1|\Lambda}$, is smooth if and only if $p_2^a|_\Lambda$, is an immersion. The consequences of this are:

(1) $p_2^a|_\Lambda$, is a closed embedding identifying Λ, (*deep space*), to a closed regular involutive manifold $V \subset \dot{T}^*Y$, (*foliation*) and
(2) $p_{1|\Lambda}$, is smooth and surjective on $\dot{T}^*X$ (*observables of space-time*).

If $c_V = \text{codim}\, V$, in T^*Y, then we have the following local model of correspondences of (?)xxxxx:

***Lemma* 4.1.** *Assume that $S \to X \times Y$, is closed embedding and assumptions given by a), and b) above. Then, for every $(p, q^a) \in \Lambda$, there exist open subsets $U_X, U'_X \subset \dot{T}^*Y$, with $p \in U_X$, and $q \in U_Y$, a complex manifold Z of dimension c_V, and a contact transformation $\psi : U_Y \overset{\cong}{\to} U'_X \times T^*Z$, such that $id_{UX} \times \psi$, induces an isomorphism of correspondences:*

$$\begin{array}{ccccc} & \Lambda\cap(U_X\times U_Y) & & \\ p1 & \swarrow \quad \searrow & P_2^a \\ U_X & & V\cap U_Y \end{array} \quad \overset{\cong}{\to} \quad \begin{array}{ccccc} & \Lambda_\chi\times T_Z^*Z & & \\ p1 & \swarrow \quad \searrow & P23^a \\ U_X & & U'_X\times T_Z^*Z \end{array} \tag{4.2}$$

where $\Lambda_\chi \subset U_X \times U'_a{}_X$, is the graph of a contact transformation χ: $U_X \overset{\cong}{\to} U'_X$, and p^a_{23}, denotes the projection $U_X \times U'^a_X \times T^*Z \overset{\cong}{\to} U'_X \times T^*Z$.

Proof. [36] □

The construction of characteristic cycles of Kashiwara associates to every object $\mathcal{G}$, (*coherent sheaf on* Y), of this category, a Lagrangian cycle $C_y(\mathcal{G})$, into $\mathcal{M} = T^*Z$, which is a cone (invariant under recalling of cotangent fibers). If $\mathcal{G}$, is the structure sheaf of closed submanifold like the given by Λ, this cycle is justly the co-normal bundle and this tend to be singular. Then we can enounce the Kashiwara theorem 3. 2, also as: "The construction of a smoothed characteristic cycle give origin to an full embedding of derived categories $D^b_{RS(V)}(\mathfrak{E}_X)$ (full subcategory of objects $\mathcal{M}$, *have cohomology groups with regular singularities along* V)

into $Mod_{RS(V)}(\mathfrak{E}_X)$ (*sheaf of micro-differential formal operators on* X)"The $\mathfrak{E}_X$-modules conforms a support ring of the D_X-modules. A $\mathfrak{E}_X$-module is a ring of microdifferential operators of finite order on T^*X. Is clear that from the lemma 4.1, we can characterize the cycles propitious to determine an embedding full of derived categories that conforms the structure of our sheaves defined in bounded derived category of the full subcategory of sheaves of K-vector spaces comprising complexes with constructible cohomology. Which *are the integral transforms* that can obtain the quantized contact transformation?

Definition 4.2. (1) Let V, be a closed conic regular involutive submanifold of T^*X, and let $\mathcal{M}$, be a coherent D_X-module. We say that $\mathcal{M}$, is simple along V, if $\mathcal{M}$, can be endowed with a good filtration $\{\mathcal{M}_k\}$, such that $G\mathcal{M}|_{\dot{T}^*X}$, is locally isomorphic to O_V, as an $O_{\dot{T}^*X}$-module. We denote by $Mod_{coh}(V; O_X)$ the full subcategory of $Mod_{coh}(D_X; O_X)$, whose objects are simple along V.

Let f: $S \to X$ is a morphism, we denote by $\underline{f}!$, and $\underline{f}^{-1}$, the proper direct image and inverse image for D-modules, and we denote by $\underline{\boxtimes}$, the exterior tensor product. To $\mathcal{M} \in D^b_{RS(V)}(D_X)$, and using (?) we associate its dual

$$D'\mathcal{M} = RHom_{DX}(\mathcal{M}, D_X \otimes_{O^X} \Omega^{\otimes} X) \tag{4.3}$$

$D_X \otimes_{O^X} \Omega^{\otimes} = O_X$, where Ω_X, is the sheaf of holomorphic forms of maximal degree. We also set $\underline{D}\mathcal{M} = \underline{D}'\mathcal{M}[d_X]$. Thus $\underline{D}'\mathcal{M}$, and $\underline{D}\mathcal{M}$, belong to $D^b_{RS(V)}(D_X)$. Let $\mathcal{K}$, be a simple $D_X \times\ _Y$-module along Λ. In particular $\mathcal{K}$, is regular holonomic, and hence $\underline{D}\mathcal{K}$, is concentrated in degree zero. For $\mathcal{M} \in D^b_{RS(V)}(D_X)$, and $\mathcal{N} \in D^b_{RS(V)}(D_Y)$, we set

$$\Phi_{\mathcal{K}}\mathcal{M} = \underline{q_2}!(\mathcal{K}^L \otimes O^{X\times Y}\underline{q_1}^{-1}\mathcal{M}) \tag{4.4}$$

where q_1, and q_2, its define $X \overset{q1}{\to} X \times Y \overset{q2}{\to} Y$. also

$$\Psi_{\mathcal{K}}\mathcal{N} = \underline{q_1}!(\underline{D}\mathcal{K} \otimes^L O^{X\times Y}\underline{q_2}^{-1}\mathcal{N})[d_X - d_Y] \tag{4.5}$$

$$\Phi^j_{\mathcal{K}}\mathcal{M} = H^j\underline{\Phi}_{\mathcal{K}}\mathcal{M} \tag{4.6}$$

$$\Psi^j_{\mathcal{K}}\mathcal{N} = H^j\underline{\Psi}_{\mathcal{K}}\mathcal{N} \tag{4.7}$$

***Theorem* 4.3.** *Assume that q_1, q_2, are proper on supp ($\mathcal{K}$), and assume a) and b) from (4.1). Let $\mathcal{M}$ be a simple D_X-module along T^*X, and let $\mathcal{N}$, be a simple D_Y-module along V. Then*

1. $\underline{\Phi}^0_{\mathcal{K}}$,*and* $\underline{\Psi}^0_{\mathcal{K}}$ *send isomorphisms (isomorphism modulo flat connections, $\varphi : \mathcal{M} \to \mathcal{N}$) to isomorphism (isomorphism modulo flat connections $\psi : \mathcal{N} \to \mathcal{M}$).*
2. $\underline{\Phi}^0_{\mathcal{K}}\mathcal{M}$, *is simple along V, and $\underline{\Psi}^0_{\mathcal{K}}\mathcal{M}$, is simple along $\dot{T}X$. Moreover, $\underline{\Phi}^0_{\mathcal{K}}\mathcal{M}$, and $\underline{\Psi}^0_{\mathcal{K}}\mathcal{N}$, are flat connections for $j \neq 0$.*
3. *The natural adjunction morphisms $\mathcal{M} \to \underline{\Psi}^0_{\mathcal{K}}\underline{\Phi}^0_{\mathcal{K}}\mathcal{M}$, and $\underline{\Phi}^0_{\mathcal{K}}\underline{\Psi}^0_{\mathcal{K}}\mathcal{N} \to \mathcal{N}$ are isomorphisms modulo flat connections. In particular, the functor*

$$M_{coh}(\dot{T}^*X, O_X) \underset{\underline{\Phi}^0_{\mathcal{K}}}{\overset{\underline{\Psi}^0_{\mathcal{K}}}{\rightleftarrows}} M_{coh}(V, O_Y) \tag{4.8}$$

are quasi-inverse to each other, and thus establish and equivalence of categories. In D-modules theory the category given by $M_{coh}(V, O_Y)$, is of the simple D_X-modules to along V. This due by the support of Radon transform of the our Radon-Penrose transform that is required. The following step is to give a result of equivalences between categories that suggest the extension to the before functors to the category of the vector bundle of lines of where it is have the classification of differential operators belonging to sheaves defined in the section 2. In effect.

***Theorem* 4.4.** [36]. *With the same hypothesis as in before theorem 4. 3, assume also* $d \geq 3$. *Then with the notation of the theorem given by the section 3, then the following correspondence is an equivalence of categories*

$$Mod_{coh}(D\mathcal{L}) \underset{G\underline{\Phi}^0_{\mathcal{K}}}{\overset{\underline{\Psi}^0_{\mathcal{K}}F}{\rightleftarrows}} M_{coh}(V, O_Y) \tag{4.9}$$

Precisely this equivalences conforms the classification given in 2. 1. 1, and 2. 2. 1., though of the homogeneous vector bundles of lines. As a corollary, using these *Penrose transform*, which is of Radon type on Lagrangian manifold Λ, we can obtain that on the complex Minkowski space $\mathbb{M}$, simple $D_{\mathbb{M}}$-modules along the characteristic manifold V, of the wave equation are classified by (half) integers, the so-called *helicity* $h(k)$. The following section establishes these equivalences using geometrical additional hypothesis. $Mod_{coh}(D\mathcal{L})$, is the full subcategory of $Mod_{coh}(DX)$, whose open sets are of the $D\mathcal{L}$, type to some bundle of lines $\mathcal{L}$. In particular. If $\mathcal{N}$, is a simple D_Y-module along V, there exist an unique (up to O_X-linear isomorhisms) line bundle $\mathcal{L}$, on X, such that $\mathcal{N} \cong \underline{\Phi}^0_{\mathcal{K}} D\mathcal{L}$, in $Mod_{coh}(D_Y, O_Y)$. In other words, the above theorem says that simple D_Y-modules along V, are classified, up to flat connections, by category of holomorphic vector bundles with flat connections

$$Mod_{coh}(V, O_Y) = M(D_Y - modules/Singularities\ to\ along\ of\ the\ involutive\ manifold\ V/flat\ connection) \tag{4.10}$$

The last theorem conclude,

$$\Phi^0_{\mathcal{K}} D\mathcal{L} \leftarrow \underline{\Phi}^0_{\mathcal{K}}(\Psi^0_{\mathcal{K}}(\mathcal{N})) \rightarrow \mathcal{N} \tag{4.11}$$

However, it one is interested in calculating explicitly the image of a D_X- module associated to a line bundle, one way to do it consists in "*quantizing*" this equivalence. In the following results we will. Let $\mathcal{M}$, be a simple D_X-module along T^*X, and let $\mathcal{N}$, be a simple DY-module along V. Then $D'\mathcal{M}\underline{\boxtimes}\mathcal{N}$, is a simple $D_{X\times Y}$-module along $\dot{T}X \times V$.

Definition 4.5.

(1) Let $p \in \dot{T}^*X \times V$, and let u be a generator at p, of $(D'\mathcal{M}\underline{\boxtimes}\mathcal{N})\hat{}$, the $\mathfrak{E}'_{X\times Y}$-module associated to $D'\mathcal{M}\underline{\boxtimes}\mathcal{N}$. Denote by $\mathcal{I}$, the annihilating ideal of u in $\mathfrak{E}'_{X\times Y}$. We say that u is simple if its symbol ideal $\underline{\mathcal{I}}$, is reduced, and hence coincide with the defining ideal $\underline{\mathcal{I}}\dot{T}^*_{X\times V}$ of $\dot{T}^*X \times V$.

(2) Let $p \in \Lambda$, we say that a section $s \in Hom_{D_{X\times Y}}(D'\mathcal{M}\underline{\boxtimes}\mathcal{N}, \mathcal{K})$, is non degenerate at p, if for a simple generator u of $D'\mathcal{M}\underline{\boxtimes}\mathcal{N}$, at p, $s(u)$ is a non degenerate section of $\mathcal{K}$, at p, in the sense of [10]. (Note that locally simple modules admit simple generators, and one checks immediately that this definition does not depend on the choice of such generators).

(3) We say that is non-degenerate on Λ, if s is non-degenerate of any $p \in \Lambda$. There is a natural isomorphism

$$\alpha : Hom_{D_{X\times Y}}(D'\mathcal{M}\boxtimes\mathcal{N},\mathcal{K})) \rightarrow Hom_{D_Y}(\mathcal{N},\underline{\Phi}_{\mathcal{K}}(\mathcal{M})) \quad (4.12)$$

Hence, a sections s, defines a D_Y-linear morphism $\alpha(s):\mathcal{N}\rightarrow\underline{\Phi}_{\mathcal{K}}(M)$.

***Theorem* 4.6.** *With the above notations, it s, is non degenerate on Λ, the $\alpha(s)$, is an isomorphism modulo flat connections.*

Proof. [36], [27]. □

Observe that when Λ, is the graph of a contact transformation (and hence V, is open in T^*Y) the above result reduce to the so-called *quantized contact transformation*, in psedo-differential equations and micro-functions [36].

5. Characteristics Cycles and Equivalences

Now in the context of the generalized D-modules to the use of the Schmid-Radon transform, and finally obtain Radon-Penrose Transform, the functor

$$\underline{\Phi_S} \;+\; additional\ geometrical\ hypothesis \quad (5.1)$$

establish an equivalence between the category $M(D_X{-}modules/flat\ connection)$ and $M(D_Y{-}modules/Singularities\ to\ along\ of\ the\ involutive\ manifold\ V/flat\ connection)$ [36]. Then our moduli space that we construct is the categorization of equivalences:

$$\begin{aligned}\mathfrak{M}_{d[S/Y]} = \{&M(D_Y - modules/Singularities\ to\ along\ of\ the\\ &involutive\ manifold\ V/flat\ connection) \rightleftharpoons H^{\bullet}(\underline{U},\mathfrak{O}\ )\}\end{aligned} \quad (5.2)$$

considering the moduli space like base [36],

$$\mathfrak{M}_{d[S/X]} = \left\{M(D_X - modules/flat\ connection) \rightleftharpoons Ker(U,\square_{h(k)})\right\} \quad (5.3)$$

then the cohomology on moduli spaces is the cohomology of the space-time with an equivalence like the given in (1.12), to a more general cohomological group that the given by $H^1(\underline{U},O_{\mathbb{P}}(k))$, and whose dimension of $H^{\bullet}(\underline{U},\mathfrak{O})$, can be calculated by the intersection methods, to a complex sheaf $\mathfrak{O}$ [31], that which is of the type $O^i_{\mathbb{P}}(k)$ (*see example 2*).

An version of theorem 4. 1, foreseen in the before section, that establishes the regularity through of the transformations realized by the functor $\underline{\Phi}$, on the categories of derived D-modules adding the corresponding *cohomological groups* of zero dimension is:

***Theorem* 5.1.** *Let v, smooth and π, proper. Be (v,π): $S\rightarrow X\times Y$, a closed embedding.*

(1) If $\mathcal{M}\in D^b_{good}(D_X)$, then $\underline{\Phi}_S(\mathcal{M})$, corresponds to $D^b_{RS(V)}(D_Y)$.
(2) If $\mathcal{M}\in Mod^b_{good}(D_X)$, y $\mathcal{M}$, is locally free and of range one, then $H^0(\underline{\Phi}_S(\mathcal{M}))$, is simple to along of V, on T^*Y.

Proof. [2] □

Our space moduli that we want to characterize is this that establishes the equivalence induced by the transform one $\underline{\Phi}_S$, between the category of coherent D-Modules on X, and the category of coherent D-Modules on Y with regular singularities on V.

We define the category $Mod^b_{good}(D_X; T^*X)$, like the localization of $Mod^b_{good}(D_X)$, by the thick sub-categoy of holomorphic bundles endowed with the flat connection:

$$M_X = \{\mathcal{M} \in Mod_{good}(D_X) | char(\mathcal{M}) \subset T^* \times X\} \tag{5.4}$$

In particular, the objects of $Mod^b_{good}(D_X; T^*X)$, are the same objects of $Mod_{good}(D_X)$, and a morphism $w : \mathcal{M} \to \mathcal{M}'$, in $Mod_{good}(D_X)$, it is become in an *isomorphism* in $Mod^b_{good}(D_X; T^*X)$, if ker w, and coker w, correspond to M_X. This equivalent to says that $\mathcal{E}w : \mathfrak{E}\mathcal{M} \to \mathfrak{E}\mathcal{M}'$, is an isomorphism on T^*X. In similar form we define M_Y, and the category $Mod^b_{good}(D_Y; T^*Y)$, obtained by localization of $Mod_{RS(V)}(D_Y)$, with respect to M_Y.

Considering the cohomology groups of zero dimension of the functors $\underline{\Phi}_S$, and $\Psi_{S^\backsim}$, we get functors that we will can denote by $\Phi^0_S, and\ \Psi^0_{S^\backsim}$, respectively. In other words we obtain the images:

$$\underline{\Phi}^0_S = H^0(\underline{\Phi}_S(\mathcal{M})), and\ \underline{\Psi}^0_S \backsim = H^0(\underline{\Psi}_S(\mathcal{M})) \tag{5.5}$$

Of where using the theorem II. 1, (using the local and microlocal analysis of the before sections) its establish the functors between categories:

$$Mod_{good}(D_X) \underset{\underline{\Phi}^0_S}{\overset{\underline{\Psi}^0_S}{\leftrightarrows}} Mod_{RS(V)}(D_Y) \tag{5.6}$$

For (3.1), (5.2) and (5.3), these equivalences conforms the Moduli space [2]:

$$\begin{aligned} \mathfrak{M} = \{\mathcal{M} | \Gamma H^*_{D^X}(Hom(Mod_{good}(D_X), Mod_{RS(V)}(D_Y)) = \\ = H^0(\underline{\Phi}_S(\mathcal{M})), \forall D_X, D_Y \in M(derived\ D - modules)\} \end{aligned} \tag{5.7}$$

The additional geometrical hypothesis in the functor (5.1), comes established by the geometrical duality of Langlands [29], which says that the derived category of coherent sheaves on a moduli space $\mathfrak{M}_{flat}[^LG, C]$, where C is the complex given by

$$C : \ldots \overset{d^{j-1}}{\rightarrow} \mathfrak{E}^{j-1} \overset{d^j}{\rightarrow} \mathfrak{E}^j \overset{d^{j+1}}{\rightarrow} \mathfrak{E}^{j+1} \overset{d^{j+2}}{\rightarrow} \mathfrak{E}^{j+2} \overset{d^{j+3}}{\rightarrow} \ldots \tag{5.8}$$

is equivalent to the derived category of D-modules on the moduli space of holomorphic vector G-bundles given by $B_G(C)$ [23]. These equivalences permits to map points of $\mathfrak{M}_{flat}[^LG, C]$, to eigen-sheaf of Hecke given by $B_G(C)$ [1].

6. Results

Theorem* 6.1.** ***(F. Bulnes)*. The moduli space by this way obtained is the moduli space $\mathfrak{M}_{Flat}(^{\mathbb{L}}G, C)$, where $\mathbb{L}$, is the space of Lagrangian submanifolds, Λ, defined with C a complex of certain special sheaf of holomorphic G-bundles (eigensheaf of Hecke).*

Proof. Is necessary go from obtained co-cycles for the Radon-Penrose transform on $Mod_{good}(D_X)$, to the obtained cycles by Penrose on $Mod_{RS(V)}(D_Y)$, and viceversa. The equivalence of both spaces of co-cycles demonstrates that the functors are the given by $Hom(Mod_{good}(D_X), Mod_{RS(V)}(D_Y)$, of the moduli space (5.7). Then these functors are G-invariants and their image under $\Gamma H^{\bullet}_{D} X$, record in a moduli space of holomorphic bundles $Bund_G(C)$ [31], which is an extension of the transformed cycles by the classic Penrose transform [6]. The equivalence under the G-invariance of holomorphic bundles it must be demonstrated using a generalized Penrose transform for D-modules that are the composition of an inverse image functor and a direct image functor on the D-module side, which is foreseen by the geometrical duality of Langlands [29]. □

In electromagnetic interactions the functors that are the objects that characterize the moduli space $\mathfrak{M}_{Flat}({}^{\mathbb{L}}G, C)|_{Mgood(D_X)}$, are $\Gamma(H^{\bullet}_{SU(2)}(Hom(\pi^1(X), U(2))))$, where $\pi^1(X)$, are homotopies on the Riemannian manifold X [3], [10].

In more general context and using the flag domains that will be necessary to define quantum gravity phenomena [27], and equivalences inside of the string theory (for example hetherotic strings, D-branes and others phenomena) to give solution to extensions of the wave equation on observables of curvature, boson and fermion equations, Schmid equation [14], and classify their differential operators on the same base of vector bundles, but now through of holomorphic vector bundles that are G-invariants.

But this only cover some aspects of brane theory like as those given outside of the homogeneous space G/H, when $H = K$. For example, are acknowledges the cases to hetherotic strings that cans develop it on D-modules that are $D_{G/C(T)}$-modules when $G/C(T)$ is a flag manifold [34]. In this case the integral operators cohomology given on such complex submanifolds is also equivalent to the integral operators cohomology on submanifolds of a complex maximum torus. What happen when these flags are complex domains or their equivalents Lagrangian submanifolds in $\mathbb{F}$

Proposition 6.2. (Recillas) The equations with non-flat differential operators can be solved by the corresponding *Szegö* kernels associated with Harish-Chandra modules [11], of corresponding spherical functions on homogeneous space G/K. The *Szegö* operator complete in some points of the Lie algebra $\mathfrak{so}$(1, n+1), [24] to the Penrose transform to the case G/K, with K compact.

Proof. Some results of representation theory obtained by the seminar of representation theory of real reductive Lie groups IM/UNAM (2000-2007) [11]. □

7. One Useful Generalization of the Penrose Transform

To can establish a moduli space that can establish a resolution to the equations of the mathematical physics given by (10) and generalized coherent D-modules that are induces $D_{\mathbb{F}}$-modules by a bundle of lines $\mathbb{F}$, with complex domains that are integral submanifolds of a Khälerian manifold is necessary to use a generalized Penrose transform with the conformal invariance of the $D_{\mathbb{F}}$-modules, and that differential operators that are not flats cans write in function of conformal operators.

This last comes reflected when a piecewise linear manifold has a differential structure. This piecewise linear manifold, define a non-symmetrical component curvature, which reflect their difficult expression by conformal operators of superior degree such as Laplacian, and Dirac differential operators.

Is necessary consider some conjectures on integral geometry, that let see the construction the geometrical hypothesis that will conforms a version of Penrose transform more useful to generates quasi-conformal operators (*quasi-D-equivariant operators*) in an analytic cohomology like given by the $\overline{\partial}$ but in a *more generalized context*, that include the *Cěch*-cohomology.

Conjecture 7.1. Is required a Penrose transform more generalized to include differential operators in Hodge representative class at a complete holomorphic cohomological classes.

In other words, there is an isomorphism

$$H^{(1)}_{holomorphic}(\mathbb{D}_+, O(-2) \to H^{(0,1)})(\mathbb{D}_+, O(-2)) \tag{7.1}$$

where $H^{(0,1)})(\mathbb{D}_+, O(-2)) \cong$ *solutions* $\{\Box F(s) = 0, \forall s \in S_+\}$, where S_+, is the Stein manifold and $\mathbb{D}_+$, is a flag domain.

Conjecture 7.2. The corresponding extended class of differential operator that can be generates like generalized conformal operators are those that are of the type Gelfand-Graev-Shapiro [15].

We want to compute the analytic cohomology $H^1(Z, O)$ of a complex manifold Z.

Since solutions of differential equations on manifolds and cohomological classes are part of the bases of this generalized Penrose transform, the theory of sheaf cohomology and D-modules was perfectly suited to it for its modern study. In [10], we used this theory to generalize and study the Penrose correspondence in (3.1). More recently (4.1), in [28], studied the generalized Penrose transform between generalized flag manifolds over a complex algebraic group G, using M. Kashiwara's correspondence (see [33]) between $quasi{-}G{-}equivariant$ $D_{G/H}$-modules and some kind of representation spaces which are $(\mathfrak{g}, H)$-modules (loosely, they are complex vector spaces endowed together with an action of the Lie algebra associated to G and a action of H which are compatible in some way) when H is a closed algebraic subgroup of G.

Conjecture 7.3. We have a generalization of Penrose transform on the homogeneous space G/H, or a Radon transform on $equivariant - D - modules$ on generalized flag manifolds.

Lemma 7.4. (F. Bulnes) *We need the version of Radon transform to compute dimensions (transfom of dimensions). Their functors with Radon transform will give the equivalences on the generalization mentioned in conjecture 7.3.*

Proof. See the [13] and [10]. □

Theorem 7.5. (F. Bulnes) $\mathfrak{M}_{G-equivariant}(G/H, C^{\smile}) \cong \mathfrak{M}_{G-equivariant}(CY, C^{\wedge})$

Proof. By the Kashiwara theorem 3.1, the class given by $C(\mathcal{G})$, is a complex whose sub-class is $C(\mathcal{B})$, where $\mathcal{B}$, is a D-brane, that is to say, of D-module $\mathcal{M} \cong (\mathcal{M}o^D\mathcal{B}_{\mathbb{F}}, C\mathcal{G}))$, since, $\{Massless\ fields\ equations\} \cong \mathcal{M}o^D\mathcal{B}_{\mathbb{F}}$, then all $\mathbb{G}$, that is complex hypersurface in the space $(\mathbb{C}^{\chi})^k$, which is complex parameterized define a Khalerian moduli space $\mathfrak{M}_{\chi} \cong (\mathbb{C}^{\chi})^k/\mathbb{G})$, where χ, is the dimension of the brane space. $\mathcal{M}$, is a D-module the which is a $\mathbb{P}$-module induced by the bundle of lines $\mathbb{P}^{\wedge}$. What happen with the orbifolds? It is necessary consider the D-branes in the orbifolios like D-modules on lines bundles in $\mathbb{P}^{\smile}$. Of fact, by the theorem 6. 1, we can demonstrate that $\mathfrak{M}_H(G, C)$ under certain duality [1], is composite for objects of derived category of D-modules on $B_G(C)$. This vector bundle of lines under the same duality [1], is their wide version which isomorphic to moduli space $\mathfrak{M}_{Higgs}(G, C)$. Using a version of the Penrose transform predicted by Recillas conjecture and geometrical duality of Langlands on D-branes, it generates the isomorphism class given by $\mathfrak{M}_{Flat}({}^{\mathbb{L}}G/H, C) \cong \mathfrak{M}_H((G, C), \omega_k)$, (using Kashiwara theorem between $D_{G/H}$-modules and some $(\mathfrak{g}.K)-modules$) since there is equivalences between D-modules of $B_G(C)$, and some D-branes of $\mathfrak{M}_H(G, C)$. Then by the proposition that says that the scheme Y is projective [35], that is to say, $C(\mathcal{B})$, is a complex whose Y, manifolds (that are orbifolds of CY-manifold), are a $\mathbb{P}$-modules. The equivalence between the two moduli spaces is completed with inverse Penrose transform. □

Corollary 7.6. (F. Bulnes) Moduli space = {homology relations of the black holes with the distinct cohomological dimensions obtained by lemma 7. 4} ≅ moduli space = {given by theorem 7. 5}.

Proof. We use the Radon transform of dimensions [13], [12], given by

$$dimY = \frac{1}{d!}\int_P \prod_{i=1}^{d} d^{4|8}x_1\mathcal{L}_1(x_1) = \frac{1}{d!}\left(\int_P d^{4|8}x\mathcal{L}(x_1)\right)^d =$$

$$= \frac{(S_1)^d}{d!} = dim\Lambda(\alpha)\int_P \left\{\frac{1}{A}\prod_{j=1}^{\infty}\left\{\int_{-\infty}^{+\infty}\phi(n_j)F(n_j)\right\}dx(t)\right\} \tag{7.2}$$

where $x\mathcal{L}(xi)$ are the Lagrangian operator of instantons considered in flat space $\mathbb{P}^{4|8}$, of the corresponding supertwistor space $\mathbb{PT}$ [30]. Then by theorem 7. 5, and using the large resolution

$$\mathbb{P}^3 \to \ldots \mathbb{P}^{1+4d} \to \mathbb{P}^{2+4d} \to \mathbb{P}^{3+4d} \to \mathbb{P}^{4+4d} \to 0 \tag{7.3}$$

we obtain the equivalence between the moduli spaces. □

Some consequences and components of the space-time related under the isomorphism discussed in before theorem are those that establish the equivalences given by (1.12), with the microlocal structure of Λ, and with Floer cohomology group deduced from the cohomology group sketch by the theorem given in the search of the integral operators cohomology that enclose the Penrose transform class.

The corresponding category associated to a Calabi-Yau threefold X, would be the Fukaya-Floer category of the moduli space of unitary flat bundles over holomorphic curves in X, denote $\mathfrak{M}_{curve}(X)$. We summarize these in the following (*table 2*).

Example 7.7. The image $\underline{\Psi}_S$ + *additional geometrical hypothesis*, given by the Penrose transform that is CY threefold, can be interpreted under the additional geometrical hypothesis, using the Leray-Serre spectral sequence for $v : Y \to \mathbb{P}^1$, gives an exact sequence

TABLE 2. Penrose transform on derived sheaves.

(Coherent D-modules) $\mathcal{M}$	$\underline{\Phi}_S$+additional geometrical hypothesis	$\underline{\Psi}_S$+additional geometrical hypothesis
Manifold:	G_2-manifold,M_7	CYthreefold,X_6
SUSY Cycles:	A-submanifolds + flat bundles	Holomorphic curves + flat bundles
Invariant:	Homology group,$H_A(M)$	Fukaya category, $Fuk(\mathfrak{M}_{curve}(X))$

$$H^1(\mathbb{P}^1, v^*O_Y) \to H^1(Y, O_Y) \to H^0(\mathbb{P}^1, R^1v^*O_Y) \tag{7.4}$$

where is the image of right functor on sheaf O_Y. Then their corresponding Hodge numbers[26] (corresponding to the cohomology groups in mirror symmetry), are $h^1(O_Y) = h^2(O_Y) = 0$.

Example 7.8. Using a similar correspondence to (3.1), in a twistor context, elements of the cohomology group $H^1(\mathbb{P}\mathcal{T}', O(-2h-2))$ correspond via the Penrose transform to spacetime fields of helicity h, so that in particular a negative helicity gluon corresponds to a twistor wavefunction of weight 0. $\mathbb{P}\mathcal{T}'$, is dual orbital corresponding to $\mathbb{PM}^+$.

Example 7.9. Fix an integer $k \geq 1$, and an exact sequence

$$0 \to O_{\mathbb{P}^1}(-k) \to O_{\mathbb{P}^1}^{\otimes 2} \to O_{\mathbb{P}^1}(k) \to 0 \tag{7.5}$$

Such extensions correspond to pairs of sections $r,\ s \in H^0(\mathbb{P}^1, O_{\mathbb{P}^1}(k))$, without common zeros. If $X = X'/H$, is a relative abelian surface on $\mathbb{P}^1$, then their fibers are supersingular surfaces (D- branes of one dimension ($\cong \mathbb{P}^1 \times \mathbb{P}^1$)) and the inclusion $O_{\mathbb{P}^1}(-k) \subset O_{\mathbb{P}^1}^{\otimes 2}$, comes from the extension (7.4). The corresponding solutions are all classes of fields on Calabi-Yau manifold. The $Fuk(\mathfrak{M}_{curve}(X))$, is the space of branes that are included in the Calabi-Yau, in the hole in the 3-dimensional space.

8. Extension of Differential Operators Classification

Applying the Penrose-Schimd transform [34], [17], to the above cohomology groups from the natural scheme of derived category discussed and given in the before section, we can translates these obstruction groups [35], explained by

$$H^{\bullet}_{D^x}(Hom(Mod_{good}(D_X), Mod_{RS(V)}(D_Y)) \supset H^{\bullet}_{D^x}(\mathcal{N}, \mathcal{O}) \supset H^{\bullet}_{D^x}(\underline{\mathcal{N}}, \mathcal{O}) \tag{8.1}$$

Then the differential operators are *not flat*, [18] but by the theorems 5. 1 and 7. 5, these operators can be rewrite for the conformal differential operators under a generalized conformal structure given by vector spaces $\{V_m \subset T_mM | m \in M\}$, and that can be generated by the involutive conic submanifolds as the given in the definition 4. 2, for example to calculate Weyl curvature, (remember that the Weyl curvature is the non-symmetrical component of the general curvature tensor [12]) and all the operators used to the G-equivariant-$D_{G/H}$-modules to flag domains [14] to quantization transforms are conformal operators. To Weyl curvature it is necesary a conformal sensor sufficiently subtle that it exhibits and detects according to the theory of integiability a $obstruction\ via\ deformation\ sheaves\ of\ D_{\mathbb{P}}-modules$, is to say, that obtains the isomorphisms classes that estabishes the equivalences of the solutions to the classes that establishes the equivalences of the solutions to the classes of partial differential equations that will conform our moduli space of the field theory. A detailed classification comes given by the classification of homomorphisms of Verma modules in [20], is in the non-singular case, both integral and non-integral. The singular case can deduced from this by translation. This have been proved to flat case in certain degree of generalization by [36].

Precisely, a careful analysis shows that translation from non-singular to singular covers all homomorphisms of Verma modules. One problem that is beginner in this study via Verma modules was that only case of even dimension of the Verma module are irreducible and so there are no non-trivial invariant operators between bundles with fractional conformal weight. In the odd dimensional case, all Verma module which are not $(half-)$, integral are likewise irreducible. Those which have half-integral conformal weight give rise to precisely one invariant [18]. These include the invariant Laplacians (*see figure 3*).

Definition 8.1. (*Verma module to space-time*), Are to Lie algebra of the Lie group $SO(1, n+1)$, where elements of algebra $\mathfrak{sl}(1, n+1)$, are differential operators. This Verma module of our interests in the classification via generating differential operators by Penrose transforms in the version Radon-Penrose transform. The group $SO(1, n+1)$, helps to identify $\mathbb{R}^n \hookrightarrow Sn$ [17], [14].

9. Some Applications and New Research Developments

We basing in the scheme on Stein manifolds from a Riemannian manifold of the space-time [38], and using the generalization by Gindikin conjectures formulated in the section 7, we obtained a result given in [13]:

Theorem 9.1. ***(F. Bulnes)*** *In the integral operator cohomology $H^{\bullet}(\mathbb{M}, \mathcal{O})$, on complex manifolds the following statements are equivalent:*

(1) The open sets M_δ, and Δ_z, are G-orbits in X, and their integrals are generalized integrals to $\mathbb{M}$.

(2) Exist an integral operator T, such that $H^{\bullet}(\mathbb{M}, \mathcal{O}) \cong_T ker\{D-equations\}$.

(3) $M_\delta = \cup_{\mathbb{M}} \pi/z$, and $\Delta_z = \cup_{\Delta} \underline{z}/\underline{\pi}, H^{\bullet}(\mathbb{M}, \mathcal{O}) \cong H(\underline{U}, \rho^{-1}O(v))$.

Proof. [13] □

Consequences and particular cases:

(1) The contour integrals are generalized functionals (Huggett) [4].

(2) The complex $(n-1)-\partial$-cohomology with coefficients in a holomorphic bundle of $\mathbb{M}$, is a cohomology of hyper-lines and hyper-planes (Gindikin) [4].

Curved Case	Differential Operator	Generalization by Irreducible Verma Modules	Curvature Correction Terms	Weight Densities
Sphere S^2	$\Delta = \nabla^2$	Verma modules with dimension n ≥ 4	$f \mapsto \nabla_b[\nabla^b\nabla^a] - 4P^{ab} - 2Pg^{ab}]\nabla_a f$	–4
Not exist in general [30]	$\nabla^3 \cong \Delta^2\nabla_A{}^{A'}: \mathcal{O}_{A'}[1] \to \mathcal{O}_A[-4]$ (*inverse image of Penrose transform*)	$n \geq 4$	Not clear	–4
	$\nabla^{n/2}: \Lambda^0 \to \Lambda^1$, n, even	$n = 4$	Terms of Weyl curvature $[\nabla_a\nabla_b - \nabla_b\nabla_a]f$ on twistor T	–4
$\wedge^2 T \otimes L^{-1}(M)$	*BGG*-Operators $\kappa_a = \Omega\kappa_a \cong \mathcal{O}_a \to (\wedge^2 T)[-1] \xrightarrow{\square} (\wedge^2 T)[-3] \to \mathcal{O}_d[-3]$ (*inverse image of Penrose transform*)	All cases $n \geq 4$ except $n \neq 5, 6$	$\begin{pmatrix} \kappa_a \\ \lambda, \theta_{bc} \\ \nu_d \end{pmatrix} \mapsto \nu_d - \frac{1}{3}\nabla_d\lambda + \frac{1}{n-5}\nabla^a\theta_{ad} + \frac{1}{(n-1)(n-6)}\Delta\kappa_d + \frac{n-8}{3(n-5)(n-6)}\nabla_d\nabla^a\kappa - \frac{(n-4)}{(n-5)(n-6)}P\kappa_d + \frac{n-8}{(n-5)(n-6)}P^2{}_d\kappa_a$	–3
On double fibration $\Delta_z \leftarrow X \to M_j$ on Flag domains	Gelfand-Shapiro-Graev Differential Operators $\kappa \circ \mathbb{B}$	All cases $n \geq 4$ except $n \neq 5$	W_{ij}, E_{ij}, and B_{ij}	
$\mathcal{T} \to M$, with group of actions $CSO(p, q)$	Conformally Invariant Operators via Curved Casimirs $C(s) = \beta(s) - 2\Sigma\phi^i \bullet (\nabla_{\xi i} - P(\xi_i)s)$	$n \geq 6$	$\sigma_a \mapsto (n-2)(n\Delta\sigma_a - 4\nabla_a\nabla^i\sigma_i)$	$n/2$, $(n-1)/2$ $w_0 = -1 = 1-(n/2) = w_{00}$

FIGURE 3. Invariant Laplacians

(3) In a generalized conform structure the general integrals of line are funcionals on arcs (Bulnes) [13].

(4) The following logic implication is trivial to particle physics [30]:

$Cohomology\ of\ C\check{e}ch \rightarrow \overline{\partial} - Cohomology \rightarrow Complexification\ of\ the\ twistor\ model\ to\ space - time \rightarrow hiper - functional\ of\ C\check{e}ch$

In field theory:

(1) In QFT, into twistor picture [32], the Penrose transform gives an identification

$$H^2(\mathbb{P}^1, O(-2s-3)) = \{\textit{hyperfunctions fields} \text{ on } \mathbb{M}, \text{ of helicity } s\}$$

(2) In the source fields the Penrose transform its reduce to Conway integral [17].

(3) In measurements of observables like curvature and torsion the corresponding Radon transform on space $\mathbb{F} = \{L | L \subset \mathbb{C}^4\}$, is the John or classic Penrose transform, and their image points are used to measure the curvature through of light waves [9].

Proposition 9.2. [23]. The L^2-cohomology groups of $\mathfrak{M}_g$, de complete Kahlerians metrics are all the same cohomology groups of D'Rham of $\mathfrak{M}_g$.

A concrete application of this result establish that the moduli space of the relations between hyperbolic waves (horocycles), and the Haar measure of the group action in $SU(2,2)$, on $\mathbb{M}$, is the moduli space of the functors

$\mathfrak{M}_0 = \{D_{\mathbb{P}}(k) | \Gamma H^{\bullet}_{SU(2)}(Hom(\pi 1(X), U(2)))) = H^j(\underline{\Phi}_{\mathbb{F}}(D_{\mathbb{P}}(-k))), \forall D_X, D_Y \in M(D_{\mathbb{P}} - modules)\}$

The D-module transform of $D_{\mathbb{P}}(-k)$ is the $D_{\mathbb{M}}$-module associated to the wave equation given in (1.11), where $h(k) = -(1 + k/2)$.

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