Optimal reconstruction of derivatives of functions belongs to the Sobolev space

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The talk is dedicated to the problem of reconstruction Weyl's derivetives of 1-periodic in each variables function $f(x_1,\ldots,x_s)$.

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and almost all monographs of numerical analysis, for example,

- Bahvalov N.S., Zhidkov N.P., Kobel'kov G.M. Numerical analysis, M.: Nauka, 1987.

- Lokusievskii O.V., Gavrilov M.B. The basic of numerical analysis, M.: TOO "Yanus 1995.

- Babenko K.I. The basic of numerical analysis, M.: Nauka, 1986.

Let $s(s = 1, \ldots, s)$ be integer and r be positive number The Sobolev space $W_2^r(0,1)^s$ is the set of all integrable function $f(x) = f(x_1, ..., x_s)$ that are 1-periodic in each variable and, wich trigonometric Fourier-Lebesgue coefficients satisfy the condition

$$\sum_{(m_1,...,m_s)\in Z^s} \left|\hat{f}\,(m_1, ..., m_s)\right|^2 \left(\overline{m}_1^{2r} + ... + \overline{m}_s^{2r}\right) \leq 1,$$

where Z^s set of all vectors $(m_1, ..., m_s)$ with integer components and $\overline{m}_j = \max\{1; \left|m_j\right|\}$ $(j = 1, .., s)$.

We consider the problem of numerical differentiation in following sense

$$\delta_N(\varepsilon_N) =$$

$$= \inf_{\xi_1,\ldots,\xi_N \in [0,1]^s, \varphi_N} \sup_{\substack{f \in W_2^r(0,1)^s \\ |f(\xi_j) - z_j| \le \varepsilon_N \\ j=1,\ldots,N}} \left\| f^{(\alpha_1,\ldots,\alpha_s)}(\cdot) - \varphi_N(z_1,\ldots,z_N;\cdot) \right\|_{L^2(0,1)^s}, \tag{1}$$

So, the problem devoted to optimal reconstruction derivitives of functions, which in short sense, consists of proved step by step three actions

1^0. Find $\succ\prec \delta_N(0)$ the best order of reconstruction by exact information;

2^0. Find sequences of errors $\{\tilde{\varepsilon}_N\}$ such that has a order like a $\delta_N(\tilde{\varepsilon}_N) \succ\prec \delta_N(0)$,

and

3^0. For any sequences $\eta_N \uparrow +\infty$: $\overline{\lim\limits_{N\to+\infty}} \frac{\delta_N(\tilde{\varepsilon}_N \eta_N)}{\delta_N(\tilde{\varepsilon}_N)} = +\infty$.

Theorem. Let $s(s = 1, 2, \dots)$ be an integer number and $r, \alpha_1, \dots, \alpha_s$ are positive numbers, such that $r > \left(\sum\limits_{i=1}^{s} \alpha_j + \frac{1}{2}\right) s$. Then

1^0. $\delta_N(0) \succ\prec N^{-\frac{r-(\alpha_1+\ldots+\alpha_s)}{s}}$;

2^0. For sequences $\tilde{\varepsilon_N} = N^{-\frac{r}{s}-\frac{1}{2}}$ satisfies the conditions :

$\delta_N(\tilde{\varepsilon}_N) \succ\prec \delta_N(0)$, Upper bound sharps on calculating aggregate

$$\varphi_N\left(f(\xi_1,\dots,\xi_N;x)\right)=$$
$$=\frac{1}{N}\sum_{\xi^{(n)}\in B_N} f(\xi^{(n)}) \sum_{|k_j|<\frac{p}{2},j=1,\dots,s} (2\pi i k_1)^{\alpha_1}\dots(2\pi i k_s)^{\alpha_s}\, e^{2\pi i(k,x-\xi^{(n)}}),$$

where

$$B_N=\left\{\xi^{(n)}=\left(\frac{n_1}{p},\dots,\frac{n_s}{p}\right):0\le n_j<p(j=1,2,\dots,s)\right\}$$

and

$$(2\pi i m_j)^{\alpha_j}:=(2\pi\left|m_j\right|)^{\alpha_j}e^{i\alpha_j\left(sgn(m_j)\frac{\pi}{2}\right)}$$

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1) Babenko K.I., *Foundations of numerical analysis*, Nauka, Moscow, 1986. (in Russian)

2) Magaril-Il'yaev G. G., Osipenko K. Yu., "Optimal recovery of functions and their derivatives from Fourier coefficients prescribed with an error Sbornik: Mathematics, 195:10 (2004), 1461-1476

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4) Magaril-Il'yaev G. G., Osipenko K. Yu., "Optimal recovery of the solution of the heat equation from inaccurate data Sbornik: Mathematics, 200:5 (2009), 665-682

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6) Plaskota L."*Noisy information and computational complexity*"Cambridge University Press (1996),P.1-308.

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